

## Evaluation of Consistency of the Knowledge in Agents

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### ABSTRACT

Given a set of agents with valid previous knowledge bases, we wish to know how new knowledge affects each agent. To model the new knowledge, boolean logic is used, expressed by 2CNF clauses, to reduce the complexity. Upon receiving new knowledge, one or more agents may find it inconsistent with their previous knowledge base, so a mechanism is applied which removes knowledge by using a contraction operation, described by the AGM model. The goal is to determine if that contradicting knowledge significantly affects the set of beliefs of each agent. Furthermore, a problem is modeled in which, given a set of agents and their knowledge base, some clauses representing new knowledge are added with the aim of determining which agent is the most affected, due to contradiction with the previous knowledge.

### RESUMEN

Dado un conjunto de agentes con una base previa de conocimiento, se desea saber cómo el nuevo conocimiento afecta a todos los agentes. Para modelar el nuevo conocimiento, se utilizó la lógica booleana, expresada mediante cláusulas 2CNF, para reducir la complejidad. Mediante la incorporación de nuevo conocimiento en los agentes, éstos podrían ser afectados de tal forma que su base de conocimiento sea inconsistente, por tanto se aplica un mecanismo, el cual elimina conocimiento mediante el uso de una operación de contracción, descrita por el modelo AGM. El objetivo es determinar si un conocimiento con contradicciones afecta significativamente al conjunto de creencias de cada agente. Por lo tanto, se modela un problema en el cual, dado un conjunto de agentes y su conocimiento base, se agregan algunas cláusulas que representan nuevo conocimiento con el objetivo de determinar cuál agente es el más afectado debido a una contradicción con el conocimiento previo. con el conocimiento previo.

Recibido: 10 de Enero del 2012

Aceptado: 14 de Febrero del 2012

### INTRODUCTION

In recent years, many formalisms have been proposed in the Artificial Intelligence literature to model common-sense reasoning. So, the revision and transformation of knowledge is widely recognized as a key problem in knowledge representation and reasoning. Reasons for the importance of this topic are the facts that intelligent systems are gradually developed and refined, and that often the environment of an intelligent system is not static but changes over time [4, 9].

Belief revision studies reasoning with changing information. Traditionally, belief revision techniques have been expressed using classical logic. Recently, the study of knowledge revision has increased since it can be applied to several areas of knowledge. Belief revision is a system that contains a corpus of beliefs which can be changed to accommodate new knowledge that may be inconsistent with any previous beliefs. Assuming the new belief is correct, as many of the previous ones should be removed as necessary so that the new belief can be incorporated into a consistent corpus. This process of adding beliefs corresponds to a non-monotonic logic [2, 8].

#### Palabras clave:

Base de conocimiento; modelo AGM; 2SAT; consistencia.

#### Keywords:

Knowledge base; AGM model; 2SAT; consistency.

The AGM (Alchourrón, Gärdenfors and Makinson) model addresses the problem of belief revision using the tools of mathematical logic [6, 13]. These works are considered the foundation for studying the problem of knowledge exchange. According to the AGM framework, knowledge  $K$  is represented by propositional logic theories and new information is represented by the same logic formulas.

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One way to represent and check the consistency of a knowledge base is modeled by using the 2SAT problem, which has been shown to be solvable in polynomial time [11]. So the knowledge of a set of agents is modeled through this formalism for the purposes of evaluating their consistency when new knowledge is added with the aim of determining which agent is the most affected due to contradiction with the previous knowledge.

The remainder of this paper is organized as follows. In Section *Preliminaries* we introduce concepts over the context of the problem. In Section *Evaluation of Knowledge Consistency* evaluates the consistency of the resulting knowledge bases for the group of agents and shows the simulation results. Finally, conclusion appear in the last section.

## PRELIMINARIES

During the 1970's from artificial intelligence and information technology the concept of "default reasoning" was introduced and defined by Raymond Reiter. This kind of logic sustains that in the absence of any contrary information, it is plausible to conclude  $X$ . It is a form of reasoning that takes into account the limitations of the agent and the commonness of things, which is pretty close to the way that everyday reasoning works. Indeed, it is due to this kind of reasoning that we can act in the world.

Well, the notion of plausible or default reasoning led to a vast area now known as non-monotonic logic or common sense, as well as circumscription logic (McCarthy), modal logic (McDermott and Doyle) and autoepistemic logic (Moore and Konolige) [4].

Non-monotonic logic is that form of reasoning under which a conclusion may be recast, retracted or defeated by an increase in information that modifies its premise. For example, the type of inference of everyday life in which people formulate tentative conclusions, reserving the right to withdraw them in light of new information. This logic satisfies the issue considering the defeatable nature of typical inferences of human common sense reasoning. Considering this type of reasoning, a formal and systematic study of cognitive processes that are present in the manipulation of knowledge structures emerges, by which an intelligent agent can draw conclusions in different ways, without having complete information to do so [5].

Before formalizing changes in beliefs, we must consider several issues: every execution of a dynamic model of beliefs must choose a language to represent

them. Whatever the chosen language, the question arises of how to represent the corpus (base) of information as well as the operations for the concepts of minimum and maximum length change of the corpus of information. This implies an epistemic theory which considers the changes in knowledge and beliefs of a rational agent. In our case, we use the criteria of rationality to determine the behavior of changes in beliefs; criteria include the minimum change of pre-existing beliefs, the primacy of new information and consistency. Thus for belief revision based on the AGM model using these criteria of rationality, three basic operations are used: expansion, contraction and review [6, 3].

Expansion is the operation that models the process of adding new knowledge to the corpus. This can be thought of as the expression of the learning process and is symbolized by the  $+$  operator, so it is defined as,  $F + p = C(F \cup p)$ , where  $F$  is the knowledge base,  $p$  is the new belief and  $C$  is the function that check new knowledge base.

Contraction is the operation that causes a new belief to remove part of the corpus of knowledge, meaning that the agent in question must stop having a certain position on this belief. This becomes complicated when there are other beliefs that would need to be abandoned based on the abandonment of the initial belief, so in the end, only the absolutely necessary beliefs would remain. This is symbolized by the operator  $-$  and is defined as  $F - p = C(F - p)$  where  $F$  is the corpus, the new belief  $p$  and  $C$  is the function that check new knowledge base.

Revision consists of modifying the set of beliefs when a new belief is incorporated into the previous set so that logical consistency is conserved. If the set of beliefs is already consistent with the new information, then the review coincides with expansion, but if new knowledge is inconsistent with any previous beliefs, the operation of review must determine the resulting set of beliefs which keeps only the part of the original which would obtain a consistent result, so the original set of beliefs must be modified by eliminating as many beliefs as necessary to ensure that the resulting set, which includes the new belief, is consistent, and is defined as  $F * p$  where  $F$  is the set of beliefs or knowledge base and  $p$  is the new belief.

To address the problems of belief revision, it is useful to consider the model using propositional logic to verify the consistency of the knowledge base in order to analyze results from adding new beliefs which are considered valid, so it is necessary to define the

concepts of propositional logic involved as follows: a formula is said to be in conjunctive normal form (CNF) if it is composed of a conjunction of disjunctive clauses and will be true if all its clauses are [1, 7].

A clause is a disjunction of literals, so that each literal stands for any formula composed of a single proposition symbol  $x$  (positive literal) or its negation  $\neg x$  (negative literal) or a constant  $\perp$  or  $\top$ .

So any formula  $F$  can be translated into an implication digraph (EF), which is a directed graph whose construction is done by taking each of the clauses  $(x_i, x_j)$  of the formula, where vertices of the graph are the  $x_i$  and  $\neg x_i$ . Here, there is a vertex for each variable and another for its negation. For each clause, two edges are generated by applying the following formula:  $(\neg x_i, x_j)$  and  $(\neg x_j, x_i)$ . The implication digraph is widely used to ensure if a formula is satisfiable or not [12].

The Satisfiability Problem (SAT) is posed as follows: given a set of variables and a constraint in conjunctive normal form, a truth assignment that satisfies the constraint must be found. In our case, we worked on CNF for 2SAT problem, which means the formula consists of clauses consisting of two literals [12].

To solve the 2SAT, the implication digraph is built and the strongly connected components of the digraph are calculated. It is said that the problem is solvable if and only if no variable and its negation belong to the same strongly connected component. There is a theorem that supports this formalism [10]:  $F$  is unsatisfiable if and only if a variable  $x$  exists such that there exist trajectories  $x$  and  $\neg x$  in  $EF$ .

## EVALUATION OF KNOWLEDGE CONSISTENCY

In artificial intelligence, there are several problems where an initial knowledge base is considered. That is the case in belief revision, which can be considered a propositional theory.

In this case, a problem is modeled to determinate the consistency of the knowledge of a group of agents whose initial knowledge base is made up of the same variables (same context). Each agent learns within the same context, so after adding a set of clauses, the affect of this new knowledge on each agent is determined.

It is therefore necessary to apply operators of expansion and contraction of knowledge to evaluate the consistency of the resulting knowledge base according to the following strategy:

## Knowledge Evaluation Strategy

First of all, it is necessary to model the knowledge base by 2-CNF to prove the validity of the knowledge of each agent. Afterwards, inconsistencies are searched for in each knowledge base. In order for new learning to take place, the base knowledge must be satisfiable, meaning that at least the positive or negative literal of each variable must be consistent.

So, the same new knowledge is gradually added to each agent's knowledge base, meaning that all agents must adjust to the new knowledge based on their previous beliefs. Each new clause represents a new belief, so each agent must analyze if the new knowledge affects their knowledge base or not. If an agent's knowledge base is not affected, it will increase, with the new knowledge, representing the assimilation of the new knowledge without contradictions.

Otherwise, if an agent's knowledge base is unsatisfiable, then that agent will have to remove the previous knowledge that generates contradictions with the new knowledge. In this case, an exhaustive search will be done to remove an indefinite number of clauses.

This process will execute as many times as necessary until the knowledge base of each agent is not fed.

## Evaluation Strategy

Input:

- A set of  $n$  agents with  $m$  clauses  $(x_i, x_j)$  that make up the satisfiable knowledge base  $F_i$ , where  $i = 1 \dots n$ .
- The new knowledge  $C_j$ , with  $j = 1 \dots T$ , where  $T$  is the number of new beliefs to be added.

For each new belief  $C_j$  For each base  $F_i$

1. Obtain the extended formula  $EF_i$  using equation (1) below:

$$EF = \{(-x_1 \vee x_2) \wedge (-x_2 \vee x_1) \wedge (-x_i \vee x_j) \wedge (-x_j \vee x_i) \dots (-x_m \vee x_n) \wedge (-x_n \vee x_m)\} \quad (1)$$

2. Create the linked list  $L$  to store the implication graph of  $EF_i$ .
3. Calculate the consistency sets  $TX$  for each literal.  
 $TX[x_i] = x_i, L[x_i] \cup L[L[x_i]]$  for each  $L[x_i]$  that does not belong to the set.  $x_i$  is said to be inconsistent if in all of  $TX[x_i]$  there is both a variable  $x_j$  and its negation  $\neg x_j$ .

4. Verify the consistency of the knowledge base  $F_i$ . If in the calculation of the set  $TX$ , some  $x_i$ ,  $TX[x_i]$  is inconsistent and  $T[-x_i]$  is also inconsistent, then the base  $F_i$  is unsatisfiable. Otherwise, the base  $F_i$  is satisfiable.

- If the base  $F_i$  is unsatisfiable, we evaluate the new knowledge base  $F_i * C_j$ . If the result is unsatisfiable, then we apply the contraction process  $F_i - C_j$  on the knowledge base.

Results simulation

The strategy described in the previous section was applied to a group of five agents with an initial consistent knowledge base, whose clauses are as follows in tables 1 and 2:

Table 1 .

Initial knowledge base of the first three agents		
$A_1$	$A_2$	$A_3$
$(x_1, x_2)$	$(-x_1, -x_2)$	$(-x_1, x_2)$
$(x_1, x_3)$	$(x_1, -x_3)$	$(x_1, -x_3)$
$(x_1, x_4)$	$(-x_1, -x_4)$	$(x_1, x_4)$
$(x_2, x_3)$	$(-x_2, x_3)$	$(-x_2, x_3)$
$(x_2, x_4)$	$(x_2, x_4)$	$(x_2, x_4)$
$(x_3, x_4)$	$(-x_3, -x_4)$	$(x_3, -x_4)$

Table 2 .

Initial knowledge base of the last two agents	
$A_4$	$A_5$
$(-x_1, -x_4)$	$(-x_1, -x_2)$
$(-x_4, -x_3)$	$(-x_1, -x_3)$
$(-x_1, -x_2)$	$(-x_1, -x_4)$
$(x_4, x_1)$	$(-x_2, -x_3)$
$(-x_4, x_1)$	$(-x_2, -x_4)$
$(x_1, x_2)$	$(-x_3, -x_4)$

Table 3 shows the number of inconsistencies generated when new knowledge is added to each agent, as well as the sum of all inconsistencies. When the process finishes the knowledge base of each agent is as table 3 shows, so the total number of clauses is shown in fig. 1 and the final knowledge base of agents is showed in tables 4 and 5.

Table 3 .

Number of inconsistencies when new knowledge is added					
Clauses	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$
$(x_1, -x_2)$	1	4	3	3	1
$(-x_3, x_4)$	2	4	4	4	2
$(x_2, -x_4)$	3	8	4	4	3
$(-x_1, x_4)$	3	4	4	7	4
$(x_2, -x_3)$	3	4	4	4	4
$(-x_1, x_3)$	4	4	4	8	4
$(-x_2, x_4)$	4	4	4	4	4
<b>Total Incons</b>	20	32	27	34	22

To guarantee the consistency of the knowledge base of each agent, a certain number of inconsistencies was obtained, as show in fig. 2, which depicts the increase of inconsistencies with respect to the new knowledge added.



Figure 1 . Final knowledge base for each agent

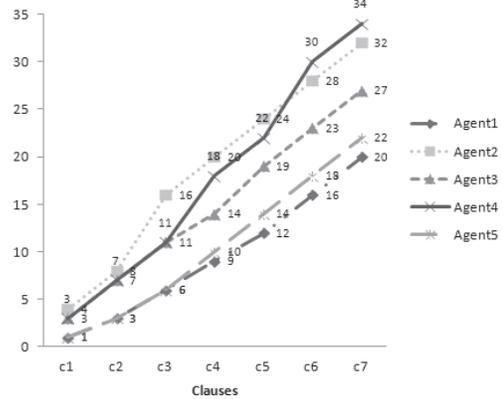


Figure 2 . Cumulative frequency of inconsistencies

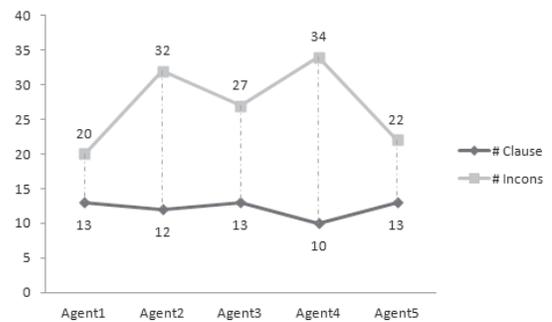


Figure 3 . Clauses vs inconsistencies

Finally, Fig. 3 shows the relationship between the clauses that make the new knowledge bases of the agents with respect to the inconsistencies generated in them.

Table 4 .

Final knowledge base of the first three agents		
A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>
(x <sub>1</sub> , x <sub>2</sub> )	(-x <sub>1</sub> , -x <sub>2</sub> )	(-x <sub>1</sub> , x <sub>2</sub> )
(x <sub>1</sub> , x <sub>3</sub> )	(x <sub>1</sub> , -x <sub>3</sub> )	(x <sub>1</sub> , -x <sub>3</sub> )
(x <sub>1</sub> , x <sub>4</sub> )	(-x <sub>1</sub> , -x <sub>4</sub> )	(x <sub>1</sub> , x <sub>4</sub> )
(x <sub>2</sub> , x <sub>3</sub> )	(-x <sub>2</sub> , x <sub>3</sub> )	(-x <sub>2</sub> , x <sub>3</sub> )
(x <sub>2</sub> , x <sub>4</sub> )	(-x <sub>3</sub> , -x <sub>4</sub> )	(x <sub>2</sub> , x <sub>4</sub> )
(x <sub>3</sub> , x <sub>4</sub> )	(x <sub>1</sub> , -x <sub>2</sub> )	(x <sub>3</sub> , -x <sub>4</sub> )
(x <sub>1</sub> - x <sub>2</sub> )	(-x <sub>3</sub> , x <sub>4</sub> )	(x <sub>1</sub> , -x <sub>2</sub> )
(-x <sub>3</sub> , x <sub>4</sub> )	(x <sub>2</sub> , -x <sub>4</sub> )	(-x <sub>3</sub> , x <sub>4</sub> )
(x <sub>2</sub> , -x <sub>4</sub> )	(-x <sub>1</sub> , x <sub>4</sub> )	(x <sub>2</sub> , -x <sub>4</sub> )
(-x <sub>1</sub> , x <sub>4</sub> )	(x <sub>2</sub> , -x <sub>3</sub> )	(-x <sub>1</sub> , x <sub>4</sub> )
(x <sub>2</sub> , -x <sub>3</sub> )	(-x <sub>1</sub> , x <sub>3</sub> )	(x <sub>2</sub> , -x <sub>3</sub> )
(-x <sub>1</sub> , x <sub>3</sub> )	(-x <sub>2</sub> , x <sub>4</sub> )	(-x <sub>1</sub> , x <sub>3</sub> )
(-x <sub>2</sub> , x <sub>4</sub> )		(-x <sub>2</sub> , x <sub>4</sub> )

Table 6 shows different instances of randomly generated tests, using a greater number of clauses and variables. In each case, the total number of eliminated clauses and the time required to guarantee knowledge base consistency are shown and on table 7 the time in seconds for the satisfiability of clauses is shown according to figure 4.

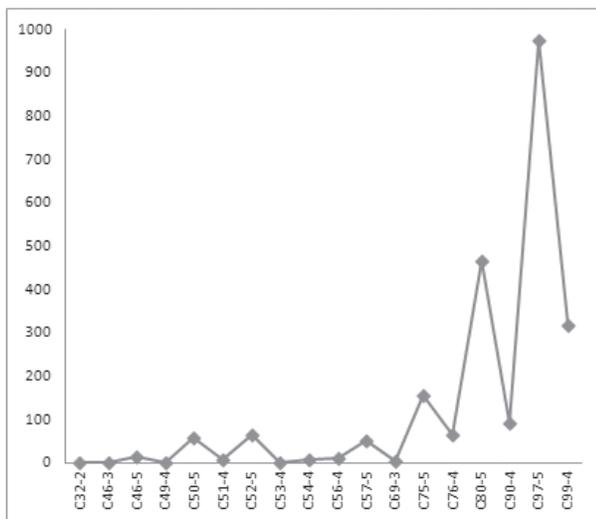


Figure 4 . time in seconds for the satisfiability of clauses

Table 6 .

Instances of randomly generated tests			
#Vars	#Clauses	#Clauses removed	Time(sec)
10	32	2	0
10	46	3	0
10	46	5	14
10	49	4	3
10	50	5	60
10	51	4	7
10	52	5	66
10	53	4	3
10	54	4	8
10	56	4	11
10	57	5	51
20	69	3	4
20	75	5	156
20	76	4	66
20	80	5	466
30	90	4	92
30	97	5	976
30	99	4	317

Table 7 .

Clauses CN-M vs time, where N is total clauses and m clauses removed	
Clauses	Time(sec)
C32 - 2	0
C46 - 3	0
C46 - 5	14
C49 - 4	3
C50 - 5	60
C51 - 4	7
C52 - 5	66
C53 - 4	3
C54 - 4	8
C56 - 4	11
C57 - 5	51
C69 - 3	4
C75 - 5	156
C76 - 4	66
C80 - 5	466
C90 - 4	92
C97 - 5	976
C99 - 4	317

## CONCLUSION

A strategy was developed to evaluate the knowledge bases of a group of agents, with the objective of guaranteeing their consistency, considering the same context.

This process resulted in determining a direct proportional relationship between the number of inconsistencies and the number of beliefs that were preserved in the knowledge base, that is, the agents with fewer clauses have a greater number of inconsistencies. Random initial knowledge bases are useful for

modelling problems involving behavior analysis of a population and guaranteeing knowledge consistency.

The final knowledge base also shows that agents with an initial monotonous knowledge base or with few contradictions do not have big changes, whereas the knowledge base of agents with a great number of contradictions tends to become reduced.

We have a simple method based on the elimination of the clause that generates the fewest inconsistencies by adding new knowledge  $p$ , this thanks to the calculation of set  $TX$  and the implication generated by the implication graph.

This type of strategy can be applied in situations where the agents involved share the same context but do not interact with one another directly. Particularly, it can be used to model social problems where the knowledge base represents a set of conflicts that an individual has, and therefore applying a new therapy or treatment (new knowledge) implies determining if it will allow for a change in behavior or not. This strategy also serves to identify the conflicting knowledge.

This type of model also has applications in disease diagnosis, in administrative decision-making or consumer habit systems, and in logical reasoning games.

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