Among quantum dots there is an interaction called Foerster interaction, it consists on the transfer of one exciton from a quantum dot to another in a non-radiative energy transfer mechanism. In this work, we develop a model of the interaction of a pair of coupled Quantum Dots (QDs), each one in its own micro cavity, interacting with its own classical field.

INTRODUCTION

For a long time scientific research in electronic systems was limited to systems as isolated atoms or particles, metals or semiconductor crystals, or beams of beta radiation; most of those are three-dimensional systems. In the early 1970s, research on electronic structures introduced an important development, quantum wells (Chang, Esaki & Tsu, 1974) (two-dimensional systems). At the beginning of the 1980s, progress in lithographic techniques allowed to confine electrons in a quasi one-dimensional structure, the so called quantum wire (Petroff, Gossard, Logan & Weigmann, 1982). Subsequent publications reported a quasi-zero dimensional structure (Cibert, Petroff, Dolan, Pearton, Gossard & English, 1986; Kash, Scherer, Worlock, Craighead & Tamargo, 1986; Reed et al., 1986), quantum dots (QDs). These structures have important and varied scientific and technological applications (Jamieson, Bakhshi, Petrova, Pocock, Imani & Seifalian, 2007; Nozik, 2002). When a quantum dot (QD) is in the presence of an electric field, there is a dipolar interaction between them, and well expected dynamics like the one with a Two Level Atom. However, if more than one QD is nearby, there is an additional quantum and non radiative coupling between the QDs, produced by the exchange of an exciton. Therefore the marriage of both interactions introduces quite an interesting dynamic that is the object of this work.

Semiclassical model of a pair of QDs

The physical system studied in this work is a pair of QDs labeled as system 1 and system 2 respectively, each one in its own cavity, see figure 1. In addition, each QD is interacting with its own classical electric field through a dipole interaction. In order to distinguish the operators corresponding to each system we will use the notation $\sigma_x^1$, $\sigma_y^1$, $\sigma_z^1$ to distinguish the Pauli’s matrices of the system 1 from the Pauli’s matrices of the system 2, $\sigma_x^2$, $\sigma_y^2$, $\sigma_z^2$. 
We will develop a model based on the Schrödinger’s picture that describes the dynamics of the coupled system.

Model

Our aim is to study a pair of QDs, each one inside its own micro cavity, interacting with their local electric field. The Hamiltonian that describes this situation is given by

\[ H = H_0 + H_j , \]

where \( H_0 \) is the free Hamiltonian:

\[
H_0 = \frac{1}{2} \hbar \left( \varepsilon_1 - W_F \right) \sigma_1^+ \sigma_1^- - hW_F \sigma_1^+ \sigma_1^- \\
+ \frac{1}{2} \hbar \left( \varepsilon_2 - W_F \right) \sigma_2^+ \sigma_2^- - hW_F \sigma_2^+ \sigma_2^- \\
- hW_F \left( \sigma_1^+ \sigma_2^- - \sigma_1^- \sigma_2^+ \right).
\]

The last term corresponds to the Foerster interaction characterized by the constant \( W_F , \varepsilon_1 \) and \( \varepsilon_2 \) the band gap energy of each QD. The interaction of the QDs with the classical electric fields \( E_1(t) \) and \( E_2(t) \) is given by \( H_j \):

\[ H_j = -d_1(t) \cdot E_1(t) \sigma_1^+ - d_2(t) \cdot E_2(t) \sigma_2^+ . \]

Where the dipoles of the quantum dot 1 and the QD 2 are given by \( d_1 \) and \( d_2 \) respectively.

Probability amplitude method

Let \( S_1 \) and \( S_2 \) represent the vector space of each system. The basis in each one of these vector spaces are \( \{|1\rangle, \{2\rangle\} \) and \( \{|3\rangle, \{4\rangle\} \) respectively. Because of the Foerster coupling, the vector space of each QD are joined making a single system in which state space is the tensor product \( S = S_1 \otimes S_2 \) of the two preceding spaces. The wave function of the QDs interaction with an electric field is given by

\[
|\psi(t)\rangle = C_1(t) \{|1\rangle, \{2\rangle\} + C_2(t) \{|1\rangle, \{4\rangle\} \\
+ C_3(t) \{|3\rangle, \{2\rangle\} + C_4(t) \{|3\rangle, \{4\rangle\} .
\]

The wave function satisfies the Schrödinger’s equation

\[ i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle . \]

We realized that the coefficients follow the differential equations

\[
\frac{d}{dt} C_1(t) = i \left( \frac{\Omega_1}{2} C_2(t) e^{-\delta t} + \frac{\Omega_2}{2} C_4(t) e^{-\delta t} \right) ,
\]

\[
\frac{d}{dt} C_2(t) = i \left( \frac{\Omega_1}{2} C_1(t) e^{-\delta t} + \frac{\Omega_2}{2} C_3(t) e^{-\delta t} + hW_F C_1(t) e^{-\delta t} \right) ,
\]

\[
\frac{d}{dt} C_3(t) = i \left( \frac{\Omega_1}{2} C_2(t) e^{-\delta t} + \frac{\Omega_2}{2} C_4(t) e^{-\delta t} + hW_F C_2(t) e^{-\delta t} \right) ,
\]

\[
\frac{d}{dt} C_4(t) = i \left( \frac{\Omega_1}{2} C_3(t) e^{-\delta t} + \frac{\Omega_2}{2} C_1(t) e^{-\delta t} \right) .
\]

RESULTS

The set of equations for the coefficients was solved analytically by using Laplace transform techniques. As a particular case we can consider that both QDs are identical and are in resonance with the frequency of their electric field. In particular, let us assume that an unapproachable QD is in the absence of field \( (\Omega_2 = 0) \).

We will carry a further analysis with experimentally sensible variables for each QD, such as the atomic inversion \( \langle \sigma_+ \rangle \) and the dipole terms for each atom \( \langle d_1 \rangle = \langle \sigma_+ \rangle + i \langle \sigma_- \rangle \) and \( \langle d_2 \rangle = \langle \sigma_+ \rangle + i \langle \sigma_- \rangle \).

We will focus our attention on one of the QDs, in order to understand the dynamics of the system in terms of the dynamics of one of the constituents of the system. So, we will introduce the normalization \( W_F = A \Omega_1 \).
The analytic equations for the dipoles of this particular system are:

\[ d_1(\tau) = -\frac{1}{2} A \exp(iA\tau) + \frac{1}{2} A \exp(-iA\tau) + \left(\frac{1}{4} + \frac{1}{4}\right) \exp(i\tau) \exp(iA\tau) - \left(\frac{1}{4} + \frac{1}{4}\right) \exp(-i\tau) \exp(-iA\tau) + \left(\frac{1}{4} - \frac{1}{4}\right) \exp(-i\tau) \exp(iA\tau) - \left(\frac{1}{4} - \frac{1}{4}\right) \exp(i\tau) \exp(-iA\tau), \]  

(7)

\[ d_2(\tau) = \frac{1}{2} A - \frac{1}{4} A \exp(i\tau) - \frac{1}{4} A \exp(-i\tau). \]  

(8)

Writing the electric dipoles in this form, allow us to explain the Fourier-transform of their oscillations.

Let us start describing system 2: it presents a three-peaked resonance fluorescence spectrum, the main peak is in the origin at the field excitation frequency of system 1 while the secondary peaks are located at Rabi frequency of system 1.

On the other hand, the first system is six-peaked distributed in pairs; each pair is shifted an amount from the peaks of the isolated system (figure 2).

For the atomic inversion of the system 1 and 2, in the weak-coupling regime \((A \ll 1)\), we have the analytical result.

\[ w_1(\tau) = \cos(\tau) \cos(A\tau) - A \sin(\tau) \sin(A\tau), \]  

(9)

\[ w_2(\tau) = -\cos(\tau) + A \sin(\tau) \sin(A\tau). \]  

(10)

In the above equations the terms contain to \(A\) as multiplicative factor are small, so the dominant part is contained in the other terms. The dominant part of system 2 shows Rabi oscillations for this system, the frequency of these oscillations is explicitly the strength of coupling \(A\). On the other hand, system 1 presents Rabi oscillations at Rabi frequency, but its oscillations are modulated by the inversion of the isolated system (figure 3).

**CONCLUSIONS**

The use of the Schroedinger’s picture allows us to find the analytical solutions of the system. We have shown results for resonant condition assuming quantum identical dots.

As a particular case we have assumed a null electric field amplitude for the isolated system. Analyzing the non-isolated system we can obtain information about the coupling. Because of the coupling there is time evolution of the population inversion of the system even in absence of electric field interactions. The inversion of the isolated system is basically oscillating at coupling frequency. The inversion of the non-isolated system is oscillating at Rabi frequency but these oscillations are modulated by the coupling frequency.

The dipole of the isolated system has oscillations which give a three-peaked spectrum. The dipole of the non-isolated system has a six-peaked spectrum. Each peak is shifted with regard to the origin and respect to
the Rabi’s frequency. These shifts are evidence of the coupling, also they give its strength.

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