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Characterization of ultrashort pulses in the focal region of refractive systems

Caracterización de pulsos ultracortos en la región focal de sistema refractivos

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ABSTRACT

In this work we analyze the spatio-temporal intensity of sub-20 fs pulses with a carrier wavelength of 810 nm along the optical axis of low numerical aperture achromatic and apochromatic doublets designed in the IR region by using the scalar diffraction theory. The diffraction integral is solved by expanding the wave number around the carrier frequency of the pulse in a Taylor series up to third order, and then the integral over the frequencies is solved by using the Gauss-Legendre quadrature method. We will show that the third-order group velocity dispersion (GVD) is not negligible for 10 fs pulses at 810 nm propagating through the low numerical aperture doublets, and its effect is more important than the propagation time difference (PTD). For sub-20 fs pulses, these two effects make the use of a pulse shaper necessary to correct for second and higher-order GVD terms and also the use of apochromatic optics to correct the PTD effect.

RESUMEN

En este trabajo analizamos la intensidad espacio-temporal de pulsos menores a 20 fs con una longitud de onda de la portadora de 810nm, a lo largo del eje óptico de dobletes acromáticos y apocromáticos de apertura numérica pequeña diseñados en la región IR, usando la teoría escalar de la difracción. La integral de difracción se plantea al expandir el número de onda alrededor de la frecuencia central del pulso en serie de Taylor hasta el tercer orden, así la integral sobre las frecuencias se resuelve usando el método de cuadraturas de Gauss-Legendre. Se muestra que la dispersión de la velocidad de grupo de tercer orden (GVD, por sus siglas en inglés) no es despreciable para pulsos menores a 10 fs y 810 nm que se propagan a través de dobletes de apertura numérica pequeña, y este efecto es más importante que la diferencia del tiempo de propagación (PTD, por sus siglas en inglés). Para pulsos menores a los 20 fs, estos dos efectos hacen necesario un modulador de pulsos para corregir el segundo orden y órdenes superiores de GVD y también el uso de óptica apocromática para corregir el efecto de PTD.

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INTRODUCTION

In recent years important progress has been made in the generation of ultrashort laser pulses. The maximum intensity is fundamental in many applications, therefore the measurement of these pulses is also fundamental. It is important that these measurements be performed with optical elements that do not change the temporal and spatial characteristics of the pulses, or, if they must, that they do so in a well-characterized manner. In many experiments pulses need to be focused on an axis to achieve high intensity, normally a lens is used for this purpose, however, lenses have chromatic and longitudinal aberrations and both may decrease the intensity due to spatial and temporal spreading of the pulse. In this context, we present an analysis of the spatio-temporal intensity of a pulse when it is focused for a doublet system along the optical axis. The diffraction integral is solved by expanding the wave number around the carrier frequency of the pulse in a Taylor series up to third order for pulses shorter than 20 fs at 810 nm. This allows

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the separation of the effects that produce the spatio-temporal spreading of the pulse: group velocity dispersion (GVD), the propagation time difference (PTD), and the aberrations of the lens (A). By using this approach we gain insight into the problem that we would not have obtained had the complete diffraction integral been solved numerically. A practical problem with the third-order expansion is that the diffraction integral over the pulse frequencies cannot be solved analytically, increasing the computational time considerably. The Gauss-Legendre quadrature method has been used to solve this computational problem, we have proved that the numerical errors in this method are negligible by taking 96 nodes and the computational time is reduced by 95% compared to the integration method by rectangles (García-Martínez, Rosete-Aguilar & Garduño-Mejía, 2012). By using the Gauss-Legendre method we evaluate the spatio-temporal intensity and we compare this quantity with the intensity reported by Bor *et al.* (Estrada-Silva, Garduño-Mejía & Rosete-Aguilar, 2011), for a simple lens with different diameters. Also we study the spatio-temporal intensity of the pulse along the axis of the achromatic doublet lens. We analyze two cases: (a) when the group velocity dispersion (GVD) is zero to all orders and (b) when only the third-order dispersion is taken into account. The notation used in the figures and tables for cases (a) and (b) are GVD=0 and GVD=1 respectively. We define the quality of the signal as being proportional to the inverse of the mean square deviations of time and space intensity distribution. The quality of the signal is then evaluated for different focal positions around the paraxial focal point of the achromatic doublet lens. With this analysis we are able to show that the third-order GVD reduces the quality of the signal for sub 20 fs pulses at 810 nm more than the propagation time difference, thus requiring the use of a pulse shaper capable of introducing second-order and higher-order dispersion correction. The achromatic lens used as example in this paper is a commercial doublet from the Edmund catalog, in the IR-region between 700 nm and 1100 nm.

Theory

We define a pulse with a carrier wavenumber, $k_0 = \omega_0/c$, where ω_0 is the optical carrier frequency and c is the speed of light in vacuum. The wavenumber is defined as $k_a = k_0(1 + \Delta\omega/\omega_0)$, where $\Delta\omega = \omega/\omega_0$ is the bandwidth around the carrier frequency ω_0. Let f_0 be the focal length of the doublet and let n_j, d_j be the refractive indices at the carrier and thicknesses for each lens of the doublet, respectively. The field distribution near the focal plane of a lens is estimated by using the scalar diffraction theory (Kempe & Rudolph, 1992), which is evaluated by expanding the wave number around the frequency of the carrier in Taylor series up to third order (Estrada-Silva *et al.*, 2011). Assuming $\Delta\omega/\omega_0 \to 0$ and following the same notation and parameter definition as in (Estrada-Silva *et al.*, 2011), the field distribution in the time domain near the focal plane of a doublet lens is given by:

$$U(x_2, y_2, z, \Delta\omega) \approx \int_{-\infty}^{\infty} dx_1 dy_1 P(x_1, y_1) U_0(x_1, y_1) A(\Delta\omega) \times e^{\Phi(x_1, y_1)} e^{-i\Theta(x_1, y_1)} e^{i\frac{k_a}{2z}[(x_2 - x_1)^2 + (y_2 - y_1)^2]}. \quad (1)$$

Where (x_1, y_1) represent the Cartesian coordinates in the lens plane, (x_2, y_2) coordinates at the focal plane, U_0 is the input field, P is the pupil function:

$$P(x_1, y_1) = \begin{cases} 1 & x_1^2 + y_1^2 \leq r_1^2 = (\rho r)^2 \text{ where } r \in [0, 1] \\ 0 & \text{otherwise} \end{cases}, \quad (2)$$

$\Theta(x_1, y_1) = K_a W(x_1, y_1)$ is the spherical aberration produced by a thin lens, $W(x_1, y_1) = 1/8S(x_1^2 + y_1^2)$, where S is the Seidel coefficient for spherical aberration, and Φ is the phase change introduced by the lens. For a doublet with refractive indices n_{01} and n_{02} for the carrier wavelength , and taking the optics coordinates, u and v and Fresnel number N as in (Kempe & Rudolph, 1992), after angular integration and by taking the expansion of the wavenumber around Taylor series, the diffraction integral is:

$$U(u, v, t) = \int_{-\infty}^{\infty} d(\Delta\omega) \int_0^1 r dr U_0(r) e^{-i\Theta(r)} e^{i(\frac{v^2}{4N})} \times e^{ik_{01}(n_{01} d_1 + n_{02} d_2)} e^{i\Delta\omega(t - \tau' - r^2\tau)} e^{(\Delta\omega)^3 (i(\gamma' - r^2\gamma)} \times e^{-(\Delta\omega)^2 [\frac{T^2}{4} - i(\delta' - r^2\delta)]} e^{i(r^{2u/2})} J_0(vr). \quad (3)$$

Where J_0 is the zero-order Bessel function of the first kind, T is half of the pulse width measured when the field amplitude falls to $1/e$. The pulse intensity full width is given by $T_{int} = \sqrt{2T}$, and $\tau', \delta', \gamma', \tau, \delta, \delta$ follow the same notation and parameter definition as in García-Martínez *et al.* (2012), where:

$$\tau \equiv \frac{k_0\rho^2}{2}\left(\frac{(n_{01}-1)b_{11}}{R_1} - \frac{(n_{01}-1)b_{11}}{R_2} + \frac{(n_{02}-1)b_{12}}{R_2} - \frac{(n_{02}-1)b_{12}}{R_3}\right) - \left(\frac{k_0\rho^2}{2\omega_0 f_0} - \frac{u}{2\omega_0}\right) = PTD + \frac{u}{2\omega_0}.$$

Coefficients a_{1j} in the expansion are calculated as follows:

$$a_{1j} \equiv \frac{1}{\omega_0} + \frac{1}{n_0}\frac{dn_j(\omega)}{d\omega}. \quad (4)$$

Where a_{2j} is the first derivative of a_{1j} and a_{3j} is the second derivative of a_{1j}. Coefficients b_{1j}, b_{2j} and b_{3j} are calculated with equation equivalent to (4) by replacing n_{0j} by $(n_{0j} - 1)$; n_{0j} is the refractive index of material j at center frequency ω_0. Let d_1 and d_2 be the thickness of each lens and R_1, R_2 and R_3 be the radii of curvature of the surfaces of the doublet. Finally, the spatially and temporally integrated quantities of $U(u, v, t)$ are determined as follows:

$$I(t) \approx \int_0^{\infty} dvv\, |U(u,v,t)|^2,\ I(v) \approx \int_{-\infty}^{\infty} dt\, |U(u,v,t)|^2. \quad (5)$$

If $i(\gamma' - r^2\gamma) = 0$, then the solution to the integral over the frequencies in (3) is analytical, but if $i(\gamma' - r^2\gamma) \neq 0$, the integral over the frequencies in (3) has to be solved numerically. We used Gauss-Legendre quadrature method to solve this integral, which reduces the numerical errors and the calculation time compared to rectangle method (García-Martínez *et al.*, 2012). From (Diels & Rudolph, 2006) a good criterion for the width distribution is given by the mean square deviation of intensity distribution:

$$<\tau_p>' = [\frac{1}{\mathcal{W}}\int_{-\infty}^{\infty} t^2 I(t)dt - \frac{1}{\mathcal{W}^2}(\int_{-\infty}^{\infty} tI(t)dt)^2]^{1/2}. \quad (6)$$

In the time domain, where $W = \int_0^{\infty} |U(u,v,t)|^2\, dt$ and the intensity is given by (5). For an unchirped Gaussian pulse the value given by (7) is normalized to one, i.e., $<\tau_p> = <\tau_p>' / T_{int}$. In a similar form the width distribution in the space domain is given by $<\tau_v>'$, and this quantity is normalized to one for a diffraction pattern at the focus of an ideal lens, that is, a lens with zero spherical aberration, zero GVD, and zero PTD (García-Martínez *et al.*, 2012), the normalized quantity is given by $<\tau_p>$. We define the quality of the signal as being proportional to the inverse of mean square deviations of time and space intensity distributions, that is:

$$S \equiv \frac{1}{<\tau_p><\tau_v>}. \quad (7)$$

The quality of the signal is equal to 1 for a pulse that suffers no temporal spreading due to GVD or PTD and suffers no spatial spreading due to the aberration of the lens.

RESULTS

A single lens made of BK7 glass, a focal length of 30 mm, and semi-diameter which varies between 0.5 mm and 8mm was used to test the quality of the signal and compare it to the intensity reported by Bor & Horváth (1994) when pulses of $T = 6$ fs are focused by a BK7 lens. In this case only PTD effect is taken into account. We can see in figure 1 that the behavior of quality of signal is the same that intensity, so we have a theoretical measured without units that describe the intensity of the signal. We assume that the second-order dispersion term is zero, i.e., $\delta = \delta' = 0$, in all the results presented in this work.

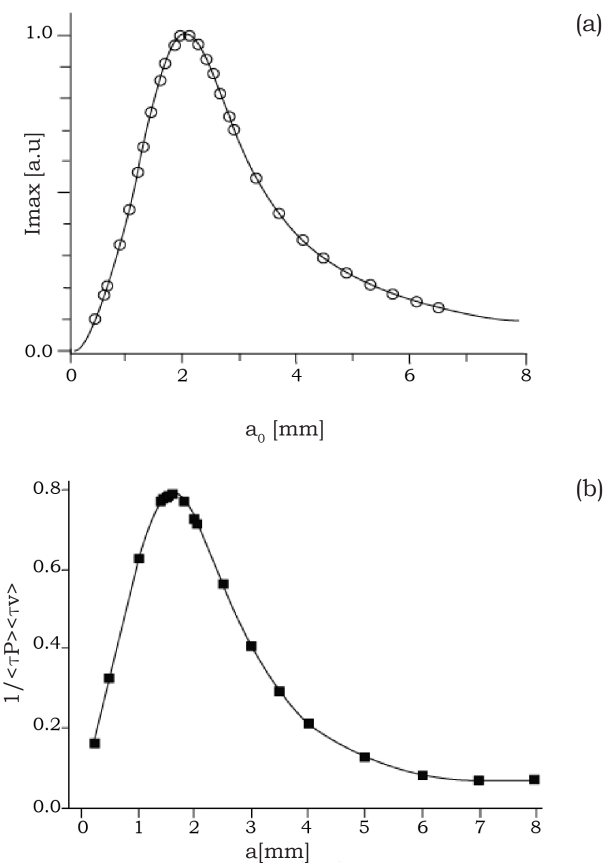
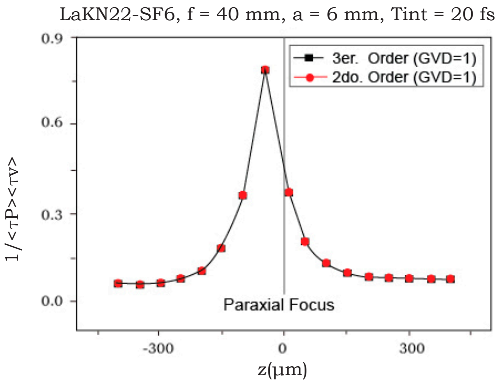


Figure 1. (a) Shows the intensity of a pulse of 6 fs, at the focus of a single lens having different semi-diameters and (b) shows the quality of the signal as a function of the semi-diameter of the lens.
Source: Authors own elaboration.

A commercial achromatic doublet, made with glasses LaK22-SF6 with a focal length of 40 mm and a diameter of 12 mm was used to analyze the focusing of pulses with an initial duration of 10 fs and 20 fs at 810 nm along the optical axis. The pulses for different positions along the optical axis between -400 mm and 400 mm around the paraxial focal point $z = 0$ mm are studied, in figure 2, we can see how the effect of GVD of third-order is more important when the duration pulse is less than 20 fs.



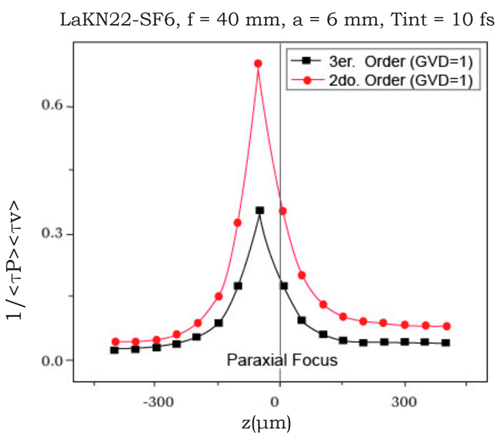


Figure 2. Third order effect of GVD decreases the quality of the signal for pulses of sub-20 fs.
Source: Authors own elaboration.

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