Acta Universitaria
DIRECCIÓN DE APOYO A LA INVESTIGACIÓN Y AL POSGRADO

Field dynamics in Layered Cylinder Resonator

Dinámica de campo en un resonador de cilindros estructurados

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ABSTRACT

We present the field dynamics in a layered cylinder resonator. The resonators can be regarded as concentric ring resonators. We found that it is possible to excite the rings with clockwise and counterclockwise modes, and these modes can be separated promoting the coupling of each of these to the output ports of the buffers of the resonators.

RESUMEN

Se presenta la dinámica de campo en un resonador de cilindros estructurados. Los resonadores pueden ser considerados como anillos concéntricos. Se encontró que es posible excitar los anillos con modos propagándose en sentido horario y antihorario, y que estos modos pueden ser separados al propiciar el acoplamiento de cada uno con diferentes puertos de la guía de onda de salida.

Recibido: 10 de noviembre de 2012
Aceptado: 14 de junio de 2013

Keywords:
Field dynamics; cylinder resonator; waveguide.

Palabras clave:
Dinámica de campo; cilindro resonador; guía de onda.

INTRODUCTION

The optical ring resonator, basically a waveguide in a closed loop, has been comprehensively studied for the past several years. Several applications have been found, including all-pass filters and add-drop filters (Heebner, Grover, Ibrahim & Ibrahim, 2007; Madsen & Lenz, 1998), optical gyroscopes (Matsko, Savchenkov, Ilchenko & Maleki, 2004), switching (Choi, Lee & Yariv, 2001; Sakhnenko, Nerukh, Benson & Sewell, 2008), etc.

Here, we revisit a variation of the optical ring resonator, conceived as a set of concentric ring resonators. It is important to note that this is not a photonic crystal fiber nor omniguide fibers (Knight, 2003; Mizrahi & Shächter, 2004; Sánchez-Mondragón, Escobedo-Alatorre, Tecpoyotl-Torres, Alvarado-Méndez, Torres-Cisneros, 2005). The resonators are layered cylinders of two refractive indices, with a matching core of a different material, which is selected in order to change the size of the resonator. The thicknesses of the layers are given by $\lambda_D/4n_{1,2}$, where λ_D is the so called "design" wavelength and $n_{1,2}$ is the refractive index in the media 1 or 2, respectively. The design wavelength is related to the Bragg frequency of the resonator by $\omega_B = 2\pi c/\lambda_D$; for a reasonably large number of layers (around 10), the field is not allowed to enter the resonator. A depiction of the resonator is shown in figure 1.

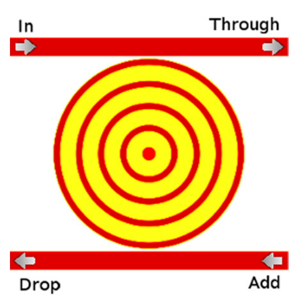

Figure 1. Waveguide-resonator setup.
Source: Author own elaboration.

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Mixed modes excitation

It is possible to excite two different modes in the resonator simultaneously. It is important, nevertheless, to keep in mind that modes in the inner rings require higher amplitudes in order to penetrate. When the field is injected to the resonator via waveguide coupling, using the setup shown in figure 1, the remaining fields in the resonator are rotating, and the rotation direction depends on the coupling coefficient. This means that it is possible to excite the resonator in a mixed mode state with each mode rotating in different direction.

An interesting application of this is mode redirection. If the resonator is excited with a pulse broad enough to span, for example, two eigenvalues of two different rings within the resonator, and the system is set such that it allows modes rotating in different directions, the fields can be coupled to a second waveguide and will propagate in different directions within that second waveguide.

This phenomena can be described by means of coupled-mode equations (Huang, 1994) in weak coupling regime. In this model, we only consider the excitation of two rings, which we can distinguish by external and internal. These equations follow:

$$\frac{da_1(t)}{dt} = \left(i\omega_1 - \frac{1}{\tau_1}\right) a_1(t) + \mu_1 b(t) + K\psi(t), \tag{1}$$

$$\frac{da_2(t)}{dt} = \left(i\omega_1 - \frac{1}{\tau_1}\right) a_2(t) + \mu_2 b(t), \tag{2}$$

$$\frac{da(t)}{dt} = \left(i\omega_2 - \frac{1}{\tau_2}\right) b(t) + \mu_1 a_1(t) + \mu_2 a_2(t), \tag{3}$$

where a_1, a_2, are the energy-amplitudes of the waves clockwise and counterclockwise respectively, propagating in the excited external ring; the clockwise propagating wave, couples to the drop port, while the counterclockwise propagating wave couples to the add port. b is the superposition of the energy-amplitude of both clockwise and counterclockwise propagating in the excited internal ring; τ_1 and τ_2 are the decay rates of each ring, μ_1 and μ_2 are the coupling coefficients of the clockwise and counterclockwise, respectively, modes with the internal ring, and ω_1, ω_2 are the resonating frequencies of the external and internal rings, respectively.

We introduce the following time scale:

$$t \to \frac{t'}{\sqrt{\tau_1 \tau_2}}, \tag{4}$$

Which implies the following frequency scales:

$$\omega_1 \to \omega_1 \sqrt{\tau_1 \tau_2}, \tag{5}$$

$$\omega_1 \to \omega_1 \sqrt{\tau_1 \tau_2}, \tag{6}$$

$$\mu_1 \to \mu_1 \sqrt{\tau_1 \tau_2}, \tag{7}$$

$$\mu_2 \to \mu_2 \sqrt{\tau_1 \tau_2}, \tag{8}$$

To compute the transmission characteristics, we consider plane wave inputs:

$$\psi(t) = A_0 e^{i\omega t}, \tag{9}$$

which leads to the following linear system:

$$i\omega a_1 = (i\omega_1 - S)a_1 + \mu_1 b + KA_0, \tag{10}$$

$$i\omega a_2 = (i\omega_1 - S)a_2 + \mu_2 b, \tag{11}$$

$$i\omega b = (i\omega_2 - S^{-1})b + \mu_1 a_1 + \mu_2 a_2, \tag{12}$$

where we have introduced the parameter $S = \sqrt{\tau_2 / \tau_1}$ as a relative decay rate. For convenience, we can also introduce the following change of variables:

$$\Omega = \frac{\omega}{\delta} - \frac{\omega_1 + \omega_2}{2\delta}, \tag{13}$$

$$\delta = \frac{\omega_2 - \omega_1}{2}, \tag{14}$$

which have a straightforward interpretation: Ω is the (normalized) frequency of the input plane wave relative to the two resonating frequencies of the rings, and δ is a measure of the separation of the resonating modes for both rings. This change of variables maps the frequencies of the resonating modes to 1 and -1. The problem reduces to solve the linear system with the following matrix:

$$\begin{bmatrix} i[\delta(\Omega+1) - iS] & 0 & -\mu_1 \\ 0 & i[\delta(\Omega+1) - iS] & -\mu_2 \\ -\mu_1 & -\mu_2 & i[\delta(\Omega-1) - iS^{-1}] \end{bmatrix}, \tag{15}$$

which leads to the following energy-amplitudes:

$$a_1 = iKA_0 \frac{\left\{\mu_2^2 + iS^{-1} + \delta(\Omega-1)\right\}\left\{\delta(\Omega+1) - iS\right\}}{\left\{\delta(\Omega+1) - iS\right\}^2 \left\{\delta(\Omega-1) - iS^{-1} + \mu^2\right\}}, \qquad (16)$$

$$a_2 = iKA_0 \frac{\mu_1 \mu_2}{\left\{\delta(\Omega+1) - iS\right\}^2 \left\{\delta(\Omega-1) - iS^{-1} + \mu^2\right\}}, \qquad (17)$$

$$\mu^2 = \mu_1^2 + \mu_2^2. \qquad (18)$$

Following the premises of the model, the transmission on the drop port is proportional to the squared modulus of a_1, given by equation (16). In the same way, transmission on the add port is proportional to the squared modulus of a_2, given by equation (17). A plot of the transmission can be seen on figure 2.

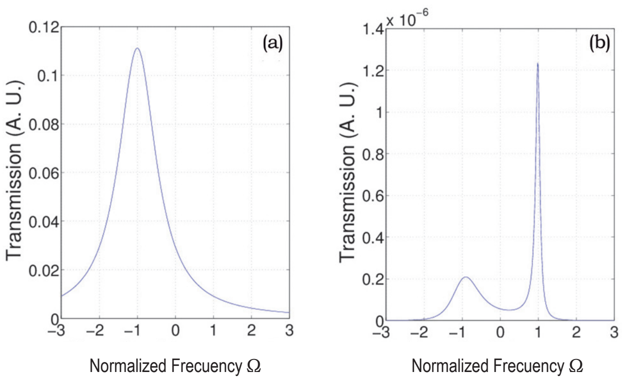

Figure 2. Transmission computed from the equations. (a): Transmission in the drop port. (b): Transmission in the add port. The following parameters were used: $S = 3$, $\delta = 5$, $\mu_1 = \mu_2 = 0.2$.
Source: Author own elaboration.

The figure shows that it is possible to couple each rotation direction of the modes to each of the ports, which allows a separation of the pulse based on its spectral components.

CONCLUSIONS

The layered resonator can be excited via waveguide coupling. In such a case, the fields remain rotating within the ring for resonant modes (which is usual in ring resonators). However, the coupling parameters can be adjusted in order to produce counter rotation in the inner rings. This can be used to separate signals with frequency width broad enough to excite two counter propagating modes in two different rings. The signals can be redirected using a second waveguide.

A model based in coupled mode theory was presented, helping in the understanding of the field dynamics within the resonator.

ACKNOWLEDGMENTS

D. L. Romero-Antequera would like to thank *Consejo Nacional de Ciencia y Tecnología* (Conacyt) for the scholarship with number 211965/223982. He also acknowledge to the *Secretaría de Educación Pública-Subsecretaría de Educación Superior-Dirección General de Educación Superior Universitaria* project 2012-01-21-002-205 and Conacyt project 000000000189688 for supporting the LAOP Workshop 2012, where this work was presented. This research was supported by Conacyt under contracts CB-2008/101378 and CB-2010-01/15786.

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