Semi classical quantum dots in their own micro cavity

Puntos cuánticos semi-clásicos en su propia micro cavidad

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ABSTRACT

Among quantum dots there is an interaction called *Foerster interaction*, it consists on the transfer of one exciton from a quantum dot to another in a non-radiative energy transfer mechanism. In this work, we develop a model of the interaction of a pair of coupled Quantum Dots (QDs), each one in its own micro cavity, interacting with its own classical field

RESUMEN

Entre los puntos cuánticos existe una interacción conocida como *interacción de Foerster,* la cual consiste en la transferencia de un excitón de un punto cuántico a otro en un mecanismo de transferencia de energía no radiativo. En este trabajo desarrollamos un modelo de la interacción de un par de puntos cuánticos acoplados (QDs por sus siglas en inglés), cada uno en su propia microcavidad, interactuando con su propio campo clásico.

INTRODUCTION

For a long time scientific research in electronic systems was limited to systems as isolated atoms or particles, metals or semiconductor crystals, or beams of beta radiation; most of those are three-dimensional systems. In the early 1970s, research on electronic structures introduced an important development, quantum wells (Chang, Esaki & Tsu, 1974) (two-dimensional systems). At the beginning of the 1980s, progress in lithographic techniques allowed to confine electrons in a quasi one-dimensional structure, the so called quantum wire (Petroff, Gossard, Logan & Weigmann, 1982). Subsequent publications reported a quasi-zero dimensional structure (Cibert, Petroff, Dolan, Pearton, Gossard & English, 1986; Kash, Scherer, Worlock, Craighead & Tamargo, 1986; Reed *et al.*, 1986), quantum dots (QDs). These structures have important and varied scientific and technological applications (Jamieson, Bakhshi, Petrova, Pocock, Imani & Seifalian, 2007; Nozik, 2002). When a quantum dot (QD) is in the presence of an electric field, there is a dipolar interaction between them, and well expected dynamics like the one with a Two Level Atom. However, if more than one QD is nearby, there is an additional quantum and non radiative coupling between the QDs, produced by the exchange of an exciton. Therefore the marriage of both interactions introduces quite an interesting dynamic that is the object of this work

Semiclassical model of a pair of QDs

The physical system studied in this work is a pair of QDs labeled as system 1 and system 2 respectively, each one in its own cavity, see figure 1. In addition, each QD is interacting with its own classical electric field through a dipole interaction. In order to distinguish the operators corresponding to each system we will use the notation σ_1^x, σ_1^y, σ_1^z to distinguish the Pauli's matrices of the system 1 from the Pauli's matrices of the system 2, σ_2^x, σ_2^y, σ_2^z.

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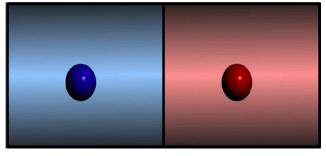
Figure 1. This picture shows the physical system studied. Each circle represents a QD in its cavity. The different color used for the cavities suggest they are interacting with different electric fields.
Source: Authors own elaboration.

We will develop a model based on the Schrödinger's picture that describes the dynamics of the coupled system.

Model

Our aim is to study a pair of QDs, each one inside its own micro cavity, interacting with their local electric field. The Hamiltonian that describes this situation is given by

$$H = H_0 + H_I, \tag{1}$$

where H_0 is the free Hamiltonian:

$$H_0 = \frac{1}{2} \hbar \left(\varepsilon_1 - W_F \right) \sigma_1^z - \hbar W_F \sigma_1^+ \sigma_1^-$$
$$+ \frac{1}{2} \hbar \left(\varepsilon_2 - W_F \right) \sigma_2^z - \hbar W_F \sigma_2^+ \sigma_2^-$$
$$- \hbar W_F \left(\sigma_1^+ \sigma_2^- - \sigma_1^- \sigma_2^+ \right). \tag{2}$$

The last term corresponds to the Foerster interaction characterized by the constant W_F, ε_1 and ε_2 the band gap energy of each QD. The interaction of the QDs with the classical electric fields $E_1(t)$ and $E_2(t)$ is given by H_I

$$H_I = -d_1(t) \cdot E_1(t) \sigma_1^x - d_2(t) \cdot E_2(t) \sigma_2^x. \tag{3}$$

Where the dipoles of the quantum dot 1 and the QD 2 are given by d_1 and d_2 respectively.

Probability amplitude method

Let S_1 and S_2 represent the vector space of each system. The basis in each one of these vector spaces are $\left| g(1), e(1) \right\rangle$ and $\left| g(2), e(2) \right\rangle$ respectively. Because of the Foerster coupling, the vector space of each QD are joined making a single system in which state space is the tensor product $S = S_1 \otimes S_2$ of the two preceding

spaces. The wave function of the QDs interaction with an electric field is given by

$$\left| \psi(t) \right\rangle = C_1(t) \left| g(1), g(2) \right\rangle + C_2(t) \left| g(1), e(2) \right\rangle$$
$$+ C_3(t) \left| e(1), g(2) \right\rangle + C_4(t) \left| e(1), e(2) \right\rangle. \tag{4}$$

The wave function satisfies the Schrödinger's equation

$$i\hbar \left| \dot{\psi}(t) \right\rangle = H \left| \psi(t) \right\rangle. \tag{5}$$

We realized that the coefficients follow the differential equations

$$\dot{c}_1(t) = i \frac{\Omega_1}{2} c_3(t) e^{-i\Delta_1 t} + i \frac{\Omega_2}{2} c_2(t) e^{-i\Delta_2 t},$$

$$\dot{c}_2(t) = i \frac{\Omega_1}{2} c_4(t) e^{-i\Delta_1 t} + i \frac{\Omega_2}{2} c_1(t) e^{-i\Delta_2 t} + \hbar W_F c_3(t) e^{-i\delta t},$$

$$\dot{c}_3(t) = i \frac{\Omega_1}{2} c_1(t) e^{i\Delta_1 t} + i \frac{\Omega_2}{2} c_4(t) e^{i\Delta_2 t} + \hbar W_F c_2(t) e^{i\delta t},$$

$$\dot{c}_4(t) = i \frac{\Omega_1}{2} c_2(t) e^{i\Delta_1 t} + i \frac{\Omega_2}{2} c_3(t) e^{i\Delta_2 t}. \tag{6}$$

To obtain the above set of equations we have done some considerations. Slow varying amplitude has been introduced, the electric fields have the explicit form $E_1(t) = e_1(t) \cos v_1 t$ and $E_2(t) = e_2(t) \cos v_2 t$ and the Rabi's frequencies are given by $\Omega_1 = d_1 \cdot e_1 / \hbar$ and $\Omega_2 = d_2 \cdot e_2 / \hbar$. QDs frequencies have been defined by $\omega_{QD1} = \varepsilon_1 - 2W_F$ and $\omega_{QD2} = \varepsilon_2 - 2W_F$, the detunings are $\Delta_1 = \omega_{QD1} - v_1$ and $\Delta_2 = \omega_{QD2} - v_2$, the difference of the QDs frequencies is denoted as $\delta = \omega_{QD1} - \omega_{QD2}$. Also we neglected the counter rotating terms, *i.e.*, Rotating Wave Approximation has been considered.

RESULTS

The set of equations for the coefficients was solved analytically by using Laplace transform techniques. As a particular case we can consider that both QDs are identical and are in resonance with the frequency of their electric field. In particular, let us assume that an unapproachable QD is in the absence of field ($\Omega_2 = 0$).

We will carry a further analysis with experimentally sensible variables for each QD, such as the atomic inversion $\langle \sigma_z \rangle$ and the dipole terms for each atom ($d_1 = \langle \sigma_1^x \rangle + i \langle \sigma_1^y \rangle$ and $d_2 = \langle \sigma_2^x \rangle + i \langle \sigma_2^y \rangle$).

We will focus our attention on one of the QDs, in order to understand the dynamics of the system in terms of the dynamics of one of the constituents of the system. So, we will introduce the normalization $W_F = A\Omega_1$.

The analytic equations for the dipoles of this particular system are:

$$d_1(\tau) = -\frac{1}{2} A \exp(iA\tau) + \frac{1}{2} A \exp(-iA\tau)$$

$$+ \left(\frac{1}{4} A + \frac{1}{4}\right) \exp(i\tau) \exp(iA\tau)$$

$$- \left(\frac{1}{4} + \frac{1}{4} A\right) \exp(-i\tau) \exp(-iA\tau)$$

$$+ \left(\frac{1}{4} A - \frac{1}{4}\right) \exp(-i\tau) \exp(iA\tau)$$

$$- \left(\frac{1}{4} A - \frac{1}{4}\right) \exp(i\tau) \exp(-iA\tau), \qquad (7)$$

$$d_2(\tau) = \frac{1}{2} A - \frac{1}{4} A \exp(i\tau) - \frac{1}{4} A \exp(-i\tau). \qquad (8)$$

Writing the electric dipoles in this form, allow us to explain the Fourier-transform of their oscillations.

Let us start describing system 2: it presents a three-peaked resonance fluorescence spectrum, the main peak is in the origin at the field excitation frequency of system 1 while the secondary peaks are located at Rabi frequency of system 1.

On the other hand, the first system is six-peaked distributed in pairs; each pair is shifted an amount from the peaks of the isolated system (figure 2).
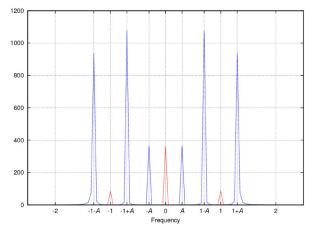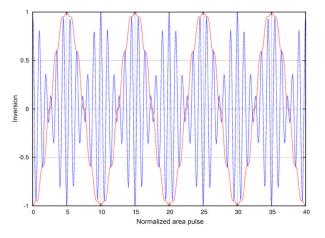

Figure 2. The resonance fluorescence spectrum of each system is shown. The blue line corresponds to the non-isolated system while the red line represents the isolated system. The isolated QD has oscillations due to the coupling and the non-isolated system has a different structure in comparison with the single QD spectrum.
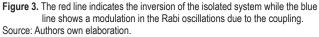Source: Authors own elaboration.

For the atomic inversion of the system 1 and 2, in the weak-coupling regime ($A \ll 1$), we have the analytical result.

$$w_1(\tau) = \cos(\tau) \cos(A\tau) - A \sin(\tau) \sin(A\tau), \qquad (9)$$

$$w_2(\tau) = -\cos(A\tau) + A \sin(\tau) \sin(A\tau). \qquad (10)$$

In the above equations the terms contain to *A* as multiplicative factor are small, so the dominant part is contained in the other terms. The dominant part of system 2 shows Rabi oscillations for this system, the frequency of these oscillations is explicitly the strength of coupling *A*. On the other hand, system 1 presents Rabi oscillations at Rabi frequency, but its oscillations are modulated by the inversion of the isolated system (figure 3).

Figure 3. The red line indicates the inversion of the isolated system while the blue line shows a modulation in the Rabi oscillations due to the coupling.
Source: Authors own elaboration.

CONCLUSIONS

The use of the Schroedinger's picture allows us to find the analytical solutions of the system. We have shown results for resonant condition assuming quantum identical dots.

As a particular case we have assumed a null electric field amplitude for the isolated system. Analyzing the non-isolated system we can obtain information about the coupling. Because of the coupling there is time evolution of the population inversion of the system even in absence of electric field interactions. The inversion of the isolated system is basically oscillating at coupling frequency. The inversion of the non-isolated system is oscillating at Rabi frequency but these oscillations are modulated by the coupling frequency.

The dipole of the isolated system has oscillations which give a three-peaked spectrum. The dipole of the non-isolated system has a six-peaked spectrum. Each peak is shifted with regard to the origin and respect to

the Rabi's frequency. These shifts are evidence of the coupling, also they give its strength.

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